

Harmonic Balance algorithm to model nonlinear effects in HTS filters subject to CDMA signals

J. Mateu, C. Collado, J. M. O'Callaghan

Abstract— We discuss an Harmonic Balance algorithm to analyze High Temperature Superconducting Circuits (HTS) subject to wideband signals, such as the ones used in CDMA wireless systems. Undersampling is used to discretize and down-convert the bandpass signal resulting from the weak HTS nonlinearities, and this has to be taken into account when performing time-domain calculations of the nonlinear effects. We present an overview of the algorithm and describe an example to verify the software developed. We also discuss a tentative performance assessment of an HTS pre-select filter in a UMTS base station receiver.

Keywords— CDMA, harmonic balance, HTS, Intermodulation, Nonlinearities, superconducting filters.

I. Introduction

Many second and third generation wireless systems use CDMA spread spectrum techniques to encode voice and data traffic and multiplex users onto the same frequency channel. Important enhancements in system performance have been obtained by using cryogenic front-ends in the base stations of these systems [1]. These front-ends include High Temperature Superconducting (HTS) pre-select filters and cooled low noise amplifiers to achieve superior sensitivity and frequency selectivity, which should make them immune to interfering signals from other operators and services. However, there is concern on the possible degradation brought by the microwave nonlinearities of the HTS materials.

HTS pre-select filters are usually made in microstrip technology and, since their nonlinear behavior is due to the properties of the HTS, the nonlinearities are distributed along the circuit pattern [2]. This is unlike most other microwave circuits whose nonlinearities are due to lumped components such as diodes and transistors. Calculation of the effects of HTS nonlinearities often requires specialized software to account for their distributed nature, and to combine it with the convoluted layouts that resonators in HTS filters usually have (see, for example [3]).

Many previous works have studied how HTS nonlinearities produce two-tone intermodulation in filters and resonators [4], [5]. Among them, we have worked on computer codes based on Harmonic Balance (HB) [6] that combine models of the local nonlinearities of HTS materials with

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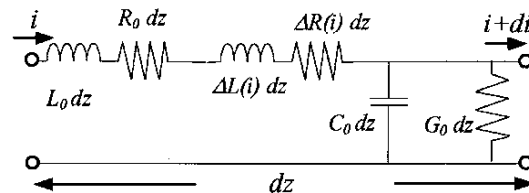


Fig. 1. Equivalent circuit valid for a small segment of HTS nonlinear transmission line of length dz , or for a small annular region of an HTS disk resonator with width dz .

circuit or electromagnetic models that account for the fact that nonlinearities are distributed [7], [8]. However, there is still very little work on how HTS devices respond to CDMA signals. Here we attempt to bridge this gap by presenting how to modify our previous algorithms to simulate the nonlinear effects produced by driving HTS devices with wideband signals such as the ones used in spread spectrum wireless systems. We refer to the resulting algorithm as Multiport Multitone Harmonic Balance (MMHB) to point out the large number of frequencies and nonlinear elements considered.

II. Multitone Multiport Harmonic Balance

Our algorithms discretize all the distributed resonators in a filter to account for spatially distributed nonlinearities. For the case of transmission line filters, this discretization consists in cascading many identical two-ports, each one modelling a segment of the transmission line much smaller than a wavelength (Fig. 1). This type of discretization is also used for modelling nonlinear effects in disk resonator filters [9]. These type of resonators are used in HTS filters when high power handling is required, since they can hold azimuthally symmetric modes having field configurations that do not generate large current densities in the HTS. Thus, the circuit in Fig. 1 can also model a small annular region of a disk resonator.

In this work we will focus on these resonators i.e., those that can be modelled as a cascade of two-ports like the one in Fig. 1. Modelling a filter with several resonators results in a large nonlinear circuit (Fig. 2), which we analyze using numerical methods based on HB [6]. In our case, the nonlinear part of the circuit would model the nonlinearities in the resistance and inductance per unit length of the transmission line or disk resonators (Fig. 1), and the linear network would model all other (linear) elements in the circuit (the linear resistance, inductance, capacitance

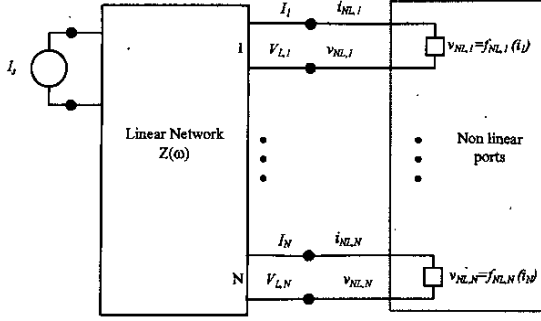


Fig. 2. Circuit to be analyzed using Harmonic Balance. There are N non-linear one-ports coming from the discretization of transmission lines or disk resonators. The linear $N+1$ port contains all the linear filter elements, including coupling between resonators, coupling to the external source, and the output load and its coupling.

and conductance per unit length of the line, the coupling between lines, etc.) [7].

The voltage v_{NL} across the N nonlinear one-ports in Fig. 2 depends on the current through them (i_{NL}) by means of a nonlinear resistance ($\Delta R(i_{NL})$) and inductance ($\Delta L(i_{NL})$). This voltage is calculated in time domain, i.e. [4]:

$$v_{NL}(i_{NL}) = \Delta R(i_{NL})i_{NL} + \frac{d}{dt} [\Delta L(i_{NL})i_{NL}]. \quad (1)$$

The linear $(N+1)$ -port in Fig. 2 is characterized in frequency domain by an impedance matrix (Z). In principle, the values of the matrix elements have to be calculated at all frequencies of the source current, and at all other frequencies where spurious signals may exist.

A. Undersampling and down-conversion of nonlinear spectra

Our procedure starts by approximating the spectrum of a wideband signal by a set of closely-spaced spectral lines close to a carrier frequency f_0 in steps of Δf ranging from $f_0 - l\Delta f$ to $f_0 + k\Delta f$:

$$s(t) \approx \text{Re} \left\{ \sum_{i=-l}^{i=k} a_i e^{j2\pi(f_0 + i\Delta f)t} \right\} \quad (2)$$

where a_i is the complex amplitude of each spectral line, and k, l are integers. The frequency resolution Δf is much smaller than f_0 and is fixed by the duration of the time window taken for the analysis of the signal ($1/\Delta f$). If this signal is subject to nonlinearities, it will result in another signal $y(t)$ with a larger number of closely spaced spectral lines grouped at odd multiples of the center frequency ($f_0, 3f_0, 5f_0, \dots$). Since nonlinearities are weak, we can neglect the spectral components about $5f_0$ and higher and $y(t)$ can be written as:

$$y(t) = \text{Re} \left\{ \sum_{q,n} b_{q,n} e^{j2\pi\kappa_{q,n}t} \right\} \quad (3)$$

with $\kappa_{q,n} = v_{q,n} + nf_0$ with $n = 1, 3$ and $v_{q,n}$ are the frequencies of the spectral lines with respect to the multiples of f_0 , that is [10]:

$$\begin{aligned} \kappa_{q,n} = & [f_0 - (2l+k)\Delta f, \dots, f_0 - \Delta f, f_0 \\ & , f_0 + \Delta f, \dots, f_0 + (2k+l)\Delta f, \\ & 3(f_0 - l\Delta f), \dots, 3f_0 - 3\Delta f, 3f_0, \\ & 3f_0 + 3\Delta f, \dots, 3(f_0 + k\Delta f)]. \end{aligned} \quad (4)$$

Sampling the spectrum of $y(t)$ using standard techniques would require a sampling rate of at least twice its maximum frequency, i.e. $6(f_0 + k\Delta f)$, resulting in an unpractically large number of samples. However, since the signal is band-limited, it is possible to downshift the signal to lower frequencies without losing information by choosing a sampling rate f_s much lower than f_0 . An analysis on how sampling $y(t)$ at f_s affects the resulting spectrum shows that a down-converted copy of the original spectrum around f_0 appears at a much lower frequency f'_0 that satisfies $f'_0 = f_0 - pf_s$, where p is an integer such that $pf_s < f_0 < (p+1)f_s$. Also, a down-converted copy of the spectrum around $3f_0$ appears at $3f'_0$. To avoid aliasing in these two down-converted spectra, the following conditions have to be satisfied:

$$f'_0 > (2l+k)\Delta f \quad (5)$$

$$f_s/2 > 3(f'_0 + k\Delta f). \quad (6)$$

Thus, to determine the sampling rate (f_s), we first find f'_0 according to (5) and then find the value of f_s that satisfies (6) and makes $(f_0 - f'_0)/f_s$ integer, so that the conditions $f'_0 = f_0 - pf_s$ and $pf_s < f_0 < (p+1)f_s$ are also fulfilled. Since CDMA signals do not have sharp spectral decays, ample margin are given to these conditions.

The down-converted version of $y(t)$ ($y_l(t)$) can be written as:

$$y_l(t) = \text{Re} \left\{ \sum_{q,n} b_{q,n} e^{j2\pi d_{q,n}t} \right\} \quad (7)$$

where $d_{q,n} = v_{q,n} + nf'_0$, i.e. the frequencies in (7) can be obtained from those listed in (4) by replacing f_0 by f'_0 .

B. Derivatives and down-converted signals

A central feature of the Harmonic Balance algorithm is the calculation of nonlinearities in time domain. In our case (see (1)), this calculation includes reactive nonlinearities whose calculation involves taking derivatives. Even though we work with down-converted signals, these derivatives should be done with the original signals which have a much faster time variations than the down-converted ones. In other words, if $y(t)$ represents the time-domain signal $\Delta L(i_{NL})i_{NL}$ in (1) and we are working with its down-converted signal $y_l(t)$ in (7), doing the nonlinear calculation in (1) involves calculating the down-conversion of dy/dt , i.e.:

$$\left. \frac{dy(t)}{dt} \right|_l = \text{Re} \left\{ \sum_{q,n} j2\pi\kappa_{q,n} b_{q,n} e^{j2\pi d_{q,n}t} \right\}. \quad (8)$$

Note that this produces a different result than taking the derivative of $y_l(t)$.

C. Impedance matrix size

One of the most restrictive limitations of MMHB is the size of the matrix Z , which depends on the number of nonlinear cells and the number of frequency components being considered [4]. Strictly speaking, Z should be defined at all frequencies of the signal source, and at the frequencies of all the spurious produced by the device, i.e. the frequencies listed in (4). However, for the case of a filter, only the components around f_0 fall within the filter passband and therefore, Z only needs to be calculated at these frequencies ($f_0 - (2l + k)\Delta f, \dots, f_0, \dots, f_0 + (2k + l)\Delta f$). In other words, the spectral components of the voltages across the nonlinear one-ports whose frequencies are significantly different from f_0 will not launch strong resonant fields in the filter that can affect the signals that we are interested in (those around f_0), and thus they can be ignored. Note that this does not change the restrictions on the sampling rate described in Sect. II-A.

D. MMHB outline

The preceding discussions result in the following outline of the MMHB method:

1. Determination of the frequency resolution Δf needed.
2. Determination of frequencies κ_{qn} : Once Δf is decided, k and l are determined to fit the bandwidth of the signal and the frequencies of the signals to be considered (κ_{qn}) are calculated using (4).
3. Calculation of the matrix Z at $f_0 - (2l + k)\Delta f, \dots, f_0, \dots, f_0 + (2k + l)\Delta f$.
4. Determination of the center frequency of the down-converted signals f_0 and sampling rate f_s .
5. Determination of the frequencies of the down-converted signals d_{qn} .
6. Start the Harmonic Balance iterative algorithm (see Fig. 3 of [4] and Fig. 2):
 - (a) Propose a solution for the current through the N nonlinear one-ports i_{NL} . The initial estimate is calculated from the circuit in Fig. 2 assuming that the voltage across the nonlinear one-ports is zero.
 - (b) With the down-converted version of i_{NL} , find the voltage across the nonlinear one-ports v_{NL} by applying (1), (7) and (8).
 - (c) Transform v_{NL} to frequency domain and find its up-converted counterpart by moving its frequency components from d_{qn} to κ_{qn} .
 - (d) With the voltage resulting from the previous point, the source current I_s , and the matrix Z , solve the linear circuit and find an updated estimate of the current through the one-ports I_{NL} .
 - (e) Down-convert I_{NL} and transform it to time domain. Compare it with i_{NL} in point (b) above. If convergence is not achieved go to point (b) and start a new iteration.

E. Verification

To verify the algorithm, we analyze the intermodulation products generated by a HTS half-wave microstrip resonator fed by two tones, both of which are within the bandpass of the resonator. In this type of resonator, the

nonlinearities in the HTS give rise to a current dependent inductance and resistance per unit length in the line ($\Delta L(i), \Delta R(i)$, see Fig. 1). We have developed equations that relate $\Delta L(i), \Delta R(i)$ with the amplitude of the intermodulation products in the resonator [11]. To verify the software developed, we have compared the result of these equations with the output of our software for the values of $\Delta L(i), \Delta R(i)$ of two different HTS samples supplied by established vendors [12] (see Annex). Note that, even though our algorithm is developed for wideband signals, it can also handle narrowband ones, like those of this verification example.

Table I shows the results of the comparison between theoretical and computed values for a two-port resonator (28.5 mm long) with a resonant frequency of 1.94 GHz, an available power of -30 dBm per tone, a frequency separation between tones of $\Delta f = 10$ Hz, and a coupling coefficient of $\beta = 4.55$. The results in this table are given in terms of the peak intermodulation current at the center of the resonator, and the agreement between simulation and theory is within 0.5 %.

	HTS ₁	HTS ₂
$I_{IMD_sim}(\mu A)$	31.84	3.85
Error(%)	< 0.2	< 0.5

TABLE I
Peak intermodulation current

III. Performance of a HTS pre-select filter in a UMTS base station

The software developed can help predict the nonlinear performance of HTS filters in base stations of wireless systems using CDMA signals. In this first tentative assessment, we have assumed an 8th order quasi-elliptic filter, similar to the one described in [13]. We have modelled the resonators as straight half-wave lines, such as the one in the verification example above. We think this might not introduce significant differences with respect to some of the resonators used in HTS filters, such as the open-loop resonator, in which the current distribution in the parts of the line close to the current maximum resembles closely that of the straight half-wave resonator.

In our equivalent circuit, couplings between resonators are modelled with ideal impedance inverters, whose values are adjusted to obtain the desired passband response over a bandwidth of 15 MHz at a central frequency of 1.9425 GHz (3 FDD UMTS uplink channels, see Fig. 3).

We have simulated a near-far scenario [14], where an ideal mobile terminal is transmitting at maximum power close to a base station of a wireless operator with an adjacent spectrum allocation (Fig. 3). We also assume that this base station is receiving -120 dBm signals from three users, each one in a 5 MHz bandwidth, filling the 15 MHz passband of the base station receiver. According to the 3GPP specifications, the interfering terminal may emit at 21 dBm output power, and the minimum path loss to the

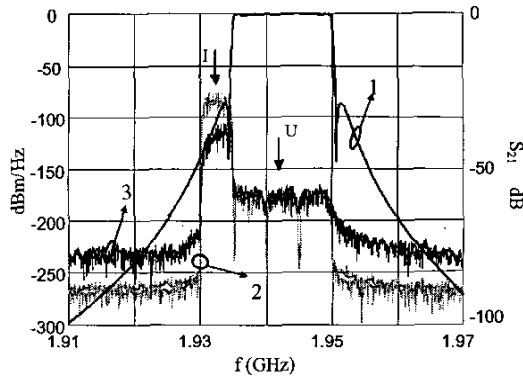


Fig. 3. Interference scenario in a UMTS FDD uplink pre-select filter with HTS materials: (1) Linear frequency response of the filter; (2) Input power spectral density; (3) Output power spectral density; (4) Interfering channel; (5) User channel being considered.

base station is 50dB, so the interfering power is -29dBm. The nonlinear effects in Fig. 3 have been calculated with sample *HTS*₁, which is the one showing stronger intermodulation distortion in Table I. The equivalent results using sample *HTS*₂ show two effects:

1. Unlike in Fig. 3, the output power outside the pass-band is significantly below the input power.
2. The power leakage from the interferer (I) to the 5MHz user channel in the center of the passband (U) (see Fig. 3) is reduced by 3.8 dB.

Both effects are due to the spectral regrowth caused by the stronger nonlinearities in sample *HTS*₁, which are noticeable despite the low power levels involved, and may cause a degradation in the traffic capacity of the base station.

Similar effects can be predicted for interferences caused by the UMTS TDD signals on this type of receiver (UMTS FDD).

IV. Conclusions

We have described the basics of the extension of our Harmonic Balance algorithms to allow them to analyze the effects of signals with many frequency components on High Temperature Superconductors. This capability is of interest since it allows to evaluate the effect of superconductor nonlinearities in filters subject to CDMA signals. The extended algorithm (MMHB) compares well with the previous ones when analyzing two-tone intermodulation, and is producing reasonable results when predicting the performance of HTS filters with 3G CDMA signals. A tentative performance assessment of a pre-select filter in a UMTS base station shows that sample quality might affect the performance of the base station, despite the low powers involved. This might raise the need for screening the nonlinear properties of HTS samples prior to filter fabrication.

V. Annex: HTS microstrip nonlinearities

We assume that the HTS material used in the resonator is YBCO as the one supplied by established vendors ($R_s = 20\mu\Omega$ at 77K). We have recently studied the nonlinearities of these type of samples [12] and found that the intermodulation properties can change significantly from sample to sample. Applying the results of [12] to a 1mm wide microstrip line over a 508 μm thick MgO dielectric ($\epsilon_r = 9.8, \tan \delta = 2 \cdot 10^{-6}$), we can predict the dependence of the inductance and resistance per unit length on the current through it and thus, the values of $\Delta R(i)$ and $\Delta L(i)$ of Fig. 1. For sample *HTS*₁, this results in $\Delta L(i) = 3.8 \times 10^{-12} |i|^{0.2}$ and $\Delta R(i) = 2.65 \times 10^{-3} |i|^{0.2}$, whereas for *HTS*₂, $\Delta L(i) = 2.2 \times 10^{-12} |i|$ and $\Delta R(i) = 2.2 \times 10^{-12} |i|$ (in all cases i is in A, ΔL is in H and ΔR in Ω).

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